Introduction

In mathematics, fractional calculus is a branch of analysis, which studies the generalization of derivation and integration from integer order n (ordinary) to non-integer order (fractional). Fractional derivation theory is a subject almost as old as classical calculus as we know it today, its origins dating back to the late 17^{th} century, the time when Isaac Newton and Gottfried Wihelm Leibniz developed the foundations of differential and integral calculus. In particular, Leibniz introduced the symbol $\frac{d^n f}{dx^n}$ to denote the derivative n^{th} of a function f when he announced in a letter to Guillaume l'Hôpital dated September 30, 1695, with the implicit assumption that $n \in \mathbb{N}$, the Hospital replied: What does the mean $\frac{d^n f}{dx^n}$ if $n = \frac{1}{2}$?. Leibniz replied: "This would lead to a paradox from which one day we will be able to draw useful conclusions. This letter from the Hospital is today accepted as the first incident of what we call fractional derivation, and the fact that the Hospital asked specifically for $n = \frac{1}{2}$, i.e. a fraction (rational number), actually gave rise to the name of this area of mathematics.

Systems described by fractional order models, using fractional differential equations based on the non-integer derivative, have attracted the interest of the scientific community. Engineers have only come to understand the importance of non-integer differential equations in the last three decades, especially when they observed that the description of some systems is more accurate than when the fractional derivative is used. Credit for the first conference is given to B. Ross who organized this conference at the University of New Haven in June 1974 under the title "Fractional Calculus and Its Applications". For the first study, another merit is attributed to K. B. Oldham and J. Spanier [6] who published a book in 1974 after a joint collaboration, started in 1968 and devoted to the presentation of the methods and applications of fractional calculus in physics and engineering . Since then, fractional calculus has gained popularity and significant consideration due mainly to the numerous applications in various fields of applied science and engineering where it has been noticed that the behavior of a large number of physical systems can be described using the fractional order derivative which provides an excellent instrument for the description of several properties of materials and processes.

In recent years, fractional differential equations have attracted the attention of many researchers due to a wide range of applications in many fields of physics, fluid mechanics, electrochemistry, viscoelasticity , nonlinear control theory, nonlinear biological systems, hydrodynamics and other fields of science and engineering. In all these scientific fields, it is important to find exact or approximate solutions to these problems. There is therefore a marked interest in the development of methods for solving fractional differential equations [1],[2],[3],[4].

The main objective of this course is:

1) Apply the basic notions of fractional calculus by designing a search for special functions.

2) Carrying out a study on integrals and fractional derivatives.

3) Use these concepts to study solutions to fractional differential equations.

This course is organized as follows: in the first chapter we present some basic theories that concern useful functions that are used in the other chapters. We give here the definitions of the Gamma, Beta and Mittag-Leffler functions. These functions play a very important role in the theory of fractional differential equations.

The second chapter is devoted to elementary definitions for integrals and fractional derivatives in the sense of Riemann-Liouville, Caputo and Grünwald-Letnikov.

The third chapter is devoted to the study of the Cauchy problem for a non-linear differential equation with Caputo fractional derivative. We demonstrate an equivalence result between this problem and a non-linear Volterra integral equation in the space of continuously differentiable functions. Based on this result, the existence and uniqueness of the solution of the considered Cauchy problem are proven.

We end this course with some references that have been used.