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Solved exercises

Exercise 1 Three light bulbs are chosen at random from 15 bulbs where 5 of them are defective. Find the probability that :
(i) none is defective
(ii) exactly one is defective
(iii) at least one is defective.
Solution :
(i) $p = \frac{C(10,3)}{C(15,5)} = \frac{120}{455} = \frac{24}{91}$.
(ii)$p = \frac{C(5,1)C(10,2)}{C(15,5)} = \frac{225}{455} = \frac{45}{91}$
(iii)$p = 1 - \frac{24}{91}$.

Exercise 2 A class contains 10 men and 20 women of which half the men and half the women have brown eyes. Find the probability that a person chosen randomly has brown eyes or is a man.
Solution
Let A be the event that the chosen person is a man, and B the event that the chosen person has brown eyes. Then

$$P(A) = \frac{10}{30} = 1/3$$
$$P(B) = \frac{15}{30} = 1/2$$
$$P(A \cap B) = \frac{5}{30} = \frac{1}{6}$$

Thus $p = P(A) + P(B) - P(A \cap B) = 2/3$

Exercise 3 A class has 12 boys and 4 girls. If three students are selected at random from the class, what is the probability that they are all boys ?
Solution
There exist $C(16,3)$ ways of selecting three students, and $C(12,3)$ ways of selecting three boys, then

$$p = \frac{C(16,3)}{C(12,3)} = \frac{11}{28}$$

Exercise 4 Ten numbered cards are there from 1 to 15, and two cards a chosen at random such that the sum of the numbers on both the cards is even. Find the probability that the chosen cards are odd-numbered.
Solution :
Let A be the event of selecting two odd-numbered cards and B the event of selecting cards whose sum is even. Then

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$
$$= \frac{C(8,2)}{C(8,2) + C(7,2)} = 4/7.$$

Exercise 5 The probability of a student passing in science is 4/5 and the of the student passing in both science and maths is 1/2. What is the probability of that student passing in maths knowing that he passed in science ?
Solution :
Let A be the event of passing in science and B the event of passing in maths. Then, probability of passing maths after passing in science is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$
$$= \frac{1/2}{4/5} = \frac{5}{8}$$

Exercise 6 Three persons A, B and C have applied for a job in a private company. The chance of their selections is in the ratio $1, 2, 4$. The probabilities that A, B and C can introduce changes to improve the profits of the company are $0.8, 0.5$ and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

Solution

Let E1, E2 and E3 respectively the events :"person A get selected", "person B get selected" and "person C get selected". Let also A the event "Changes introduced but profit not happened". Then, $P(E1) = 1/(1+2+4) = 1/7, P(E2) = 2/7$ and $P(E3) = 4/7$. We have also,

$$P(A|E1) = 1 - 0.8 = 0.2$$
$$P(A|E2) = 1 - 0.5 = 0.5$$
$$P(A|E3) = 1 - 0.3 = 0.7$$

Applying Bays' formula it follows that

$$P(E3|A) = \frac{P(A|E3)P(E3)}{P(A|E1)P(E1) + P(A|E2)P(E2) + P(A|E3)P(E3)}$$
$$= \frac{0.7\frac{4}{7}}{0.2\frac{1}{7} + 0.5\frac{2}{7} + 0.7\frac{4}{7}} = \frac{7}{10}$$

Exercise 7 A doctor is called to see a sick child. The doctor has prior information that 90% of sick children in that neighborhood have the flu, while the other 10% are sick with measles.

Let F stand for an event of a child being sick with flu and M stand for an event of a child being sick with measles. Assume for simplicity that $F \cup M = S$, i.e., that there no other maladies in that neighbourhood. A well-known symptom of measles is a rash (the event of having which we denote R). Assume that the probability of having a rash if one has measles is $P(R|M) = 0.95$. However, occasionally children with flu also develop rash, and the probability of having a rash if one has flu is $P(R|F) = 0.08$. Upon examining the child, the doctor finds a rash. What is the probability that the child has measles?

Solution

Using Bayes' formula we get

$$P(M|R) = \frac{P(R|M)P(M)}{P(R|M)P(M) + P(R|F)P(F)} \qquad = \frac{0.95 \times 0.10}{0.95 \times 0.10 + 0.08 \times 0.90} = 0.568$$

Exercise 8 Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side is colored black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Solution

Let RR, BB, and RB denote, respectively, the events that the chosen cars is the red-red, the black-black, or the red-black card. Letting R be the event that the upturned side of the chosen card is red, we have that the desired probability is obtained by

$$P(RB|R) = \frac{P(RB \cap R)}{P(R)}$$
$$= \frac{P(R|RB)P(RB)}{P(R|RR)P(RR) + P(R|RB)P(RB) + P(R|BB)P(BB)}$$
$$= \frac{1/21/3}{1 \times 1/3 + 1/2 \times 1/3 + 0 \times 1/3} = 1/3$$

Exercise 9 It is estimated that50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%.

Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

Solution

Let A the event that an email is detected as spam, and B the event that an email is spam. Hence by the Bayes's formula we have

$$\begin{aligned} P(B^c|A) &= \frac{P(B^c \cap A)}{P(A)} \\ &= \frac{P(A|B^c)P(B^c)}{P(A|B)P(B) + P(A|B^c)P(B^c)} \\ &= \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.99 \times 0.5} = 5/104. \end{aligned}$$

Exercise 10 In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumours and 90% of benign tumours.)
95% of physicians estimated the probability of cancer to be about 75%. Do you agree ?

Solution

Let M be the event "tumour is malignant", B for the "tumour is benign" and M^+ for the event "mammogram result is positive". Hence using Bayes' formula we get :

$$P(M|M^+) = \frac{P(M^+|M)P(M)}{P(M^+|M)P(M) + P(M^+|B)P(B))} \tag{1}$$

$$= \frac{0.80 \times 0.01}{0.80 \times 0.01 + 0.10 \times 0.99} = 0.075. \tag{2}$$

So the chance would be 7.5%. A far cry from a common estimate of 75%.