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Numerical methods and programming

Exercise 1 (7 pts)

Let f be a function defined on $]0, +\infty[$ by $f(x) = e^{-x} - \ln(x)$.

1- Prove that there exists a unique root ξ of the function f, located between two consecutive integers **a** and $\mathbf{a+1}$.

2- Calculate the number n of required iterations to reach an error less than 10^{-4} using Bisection method.

3- Give the iterative scheme of the Newton-Raphson algorithm.

4- Approximate the value of ξ with a precision of 10^{-4} using the Newton-Raphson algorithm, justifying the choice of the initial point x_0 between **a** and $\mathbf{a+1}$.

Exercise 2 (7 pts)

Consider the same function f of the previous exercise defined on $]0, +\infty[$ by $f(x) = e^{-x} - \ln(x)$.

1- Determine the analytical expression of $\int f(x)dx$.

2- Give the iterative scheme of the Simpson's algorithm to approximate a limited integral.

3- Using the Simpson's algorithm, approximate the value of the integral $\int_2^3 f(x)\,dx$ with a precision of 10^{-4}.

4- Using the trapezoidal method, determine the required number of subdivisions to approximate the integral $\int_2^3 f(x)\,dx$ with a precision of 10^{-4}.

Exercise 3 (7 pts)

Consider the following ordinary differential equation :

$$\frac{dy}{5y-3} = dt \tag{1}$$

1- Determine the expression of the general solution of the given ODE.

2- Give the iterative scheme of the Runge-Kutta method of order 4 (RK4) for approximating a solution of an ODE.

3- Given that $y(0) = 1$, give an approximation of $y(0.5)$ using RK2 with a steplength $h = 0.25$. Compute the error between $y(0.5)$ and its approximated value.

Remark :
- Calculators cannot be shared.
- The solutions must be written rigorously.