Setif1 University Faculty of sciences TC-SM /
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Maths1 1^{st} guided works

Exercise 1.

1 Consider the following sets

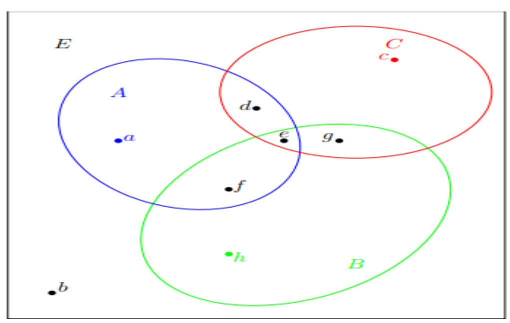
$$A = \{1,3,7,9,12\}, \quad B = \{1,3,2\}, \quad C = \{3,4,7,9\}, \quad D = \{3,1\}.$$

Describe the following sets and their cardinals: $A \cap B$, $A \setminus B$, $A \Delta B$, $D \times C$, $*B \cap C$, $\complement_D(A)$, $*D \cup A$, $\mathcal{P}(C)$.

2 Describe the following sets:

$$F = [-2, 1[\cap] - \infty, 0], *E = [-2, 1[\cup] - \infty, 0], *G = [-2, 1[\Delta] - \infty, 0], *H = \complement_{\mathbb{R}}(F).$$

Exercise 2: Consider the following diagram, witch contains three subsets A, B, C of a set E and the elements a, b, c, d, e, f, g, h of E



Determine whether the following statements are true or false
1) $g \in A \cap \bar{B}$ 2) $g \in \bar{A} \cap \bar{B}$ 3) $g \in \bar{A} \cup \bar{B}$
4) $f \in \bar{A}$ 5) $e \in \bar{A} \cap \bar{B} \cap \bar{C}$. 6) $\{h, b\} \subset \bar{A} \cap \bar{B}$
7) $\{a, f\} \subset A \cup C$.

Exercise 3: If we have $C \subset A \cup B$, is it because $C \subset A$ or $C \subset B$?

Exercise 4: Let A, B, C be three subsets of the set E, for $X \subset E$, denoted by X^c the complement of X in E.
Prove the following Morgan's laws :
1. $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ *2. $(A^c)^c = A$.

Exercise 5 : *Find the set of parts of the set $E = \{a, b, c, d\}$.

Exercise 6 : Let E and F be two sets, and let A and C be subsets of E and D, B be two subsets of F.Prove it

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$

Exercise 7: Determine if the relations are reflexive, symmetric, transitive, or antisymmetric.
(1) $E = \mathbb{Z}$ and $x\mathcal{R}y \Leftrightarrow x = -y$
*(2) $E = \mathbb{R}$ and $x\mathcal{R}y \Leftrightarrow \cos^2 x + \sin^2 y = 1$
*(3) $E = \mathbb{N}$ and $x\mathcal{R}y \Leftrightarrow \exists p, q \geq 1,\ y = px^q$. Where p and q are natural numbers.

Exercise 8: Let $\mathcal{R}$ be the relation defined on $\mathbb{R}^2$ by: $(x_1, y_1)\mathcal{R}(x_2, y_2) \Leftrightarrow y_1 = y_2$.
(1) Show that $\mathcal{R}$ is an equivalence relation.
(2) Determine the equivalence class of $(1, 0)$.
*(3) Same questions for the relation $\mathcal{R}$ defined on $\mathbb{R}^2$ by: $(x_1, y_1)\mathcal{R}(x_2, y_2) \Leftrightarrow x_1^2+y_1^2 = x_2^2+y_2^2$.

Exercise 9: Let $\mathcal{R}$ be the relation defined on $\mathbb{N}^*$ by : $n\mathcal{R}m \Leftrightarrow \exists k \in \mathbb{N}^* : n = km$..
(1) Show that $\mathcal{R}$ is a relation of order
(2) Is the order total?

Exercise 10:

1. Let the function be $f : \mathbb{R} \to \mathbb{R}$, where $x \mapsto x^2$and let be.$A = [-1, 4]$ find :
   - The direct image of the set A by application f .
   - The reciprocal (inverse) image of the set A by application f.
2. *Let the function be $\sin : \mathbb{R} \to \mathbb{R}$.
   - What is the direct image of the set $\mathbb{R}$, the set $[0, 2\pi]$ and the set $[0, \pi/2]$?
   - What is the inverse image of set $[0, 1]$, the set $[3, 4]$ and the set $[1, 2]$?

Exercise 11: Show that the following functions are maps and then check whether they are injective, surjective or bijective
$f_1 : \mathbb{Z} \to \mathbb{Z},\ n \mapsto 2n,$ $\quad f_2 : \mathbb{Z} \to \mathbb{N},\ n \mapsto 4n^2 + 5$
$f_3 : \mathbb{R} \to \mathbb{R},\ x \mapsto x^2,$ $\quad f_4 : \mathbb{R} \to \mathbb{R}+, x \mapsto x^2.$

Exercise 12: Let the functions f and g defined from $\mathbb{N}$ to $\mathbb{N}$ by the following:

$$f(x) = 2x \text{ and } g(x) = \begin{cases} \dfrac{x}{2}, & \text{If } x \text{ is even} \\ 0, & \text{If } x \text{ is odd} \end{cases}$$

Find $g \circ f$ and $f \circ g$. Are the functions f and g injective, surjective or bijective?

Exercise 13: Show by recurrence

1. For any $n \in \mathbb{N} : 5n^3 + n$ is divisible by 6. (Indication $n\,(n + 1) = 2p,\ p \in \mathbb{N}$).
2. *For any $n \in \mathbb{N} : 2^n > n$.
3. *If $x \neq 1,\ x \in \mathbb{R},\ n \geq 1 : 1 + x + x^2 + ... + x^{n-1} = \dfrac{1 - x^n}{1 - x}$.

   Instruction. The questions mentioned by (*) are left to the students.