U.F.A.S1. University **Department of Mathematics**
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Sheet 2 Maths 1 [1]

Exercice 1 :

Determine the definition domain for each function f, g, $f \circ g$ and $g \circ f$ with :

1. $f(x) = x^2 - 4, \quad g(x) = \sqrt{x}$
2. $f(x) = \sqrt{x}, \quad g(x) = \dfrac{x-1}{x-2}$

Exercice 2 :

Determine the lower bound, the upper bound, the infimum, the supremum, the maximum and the minimum for each set if they exist :

$$A = \{x \in \mathbb{R} : x^2 < 2\}, \quad B = \left\{\frac{1}{n};\, n \in \mathbb{N}^*\right\}, \quad C = \left\{\frac{1}{n} + (-1)^n;\, n \in \mathbb{N}^*\right\},$$

$$D = \left\{\frac{2n}{2n-1};\, n \in \mathbb{N}^*\right\} \qquad (*), \quad E = \left\{\frac{x+1}{x+2};\, x \leq -3\right\}.$$

Exercice 3 :

The same questions for the following sets :

$$X = \left\{\frac{2xy}{x^2+y^2};\, x \in \mathbb{R}^*,\, y \in \mathbb{R}^*\right\}, \quad Y = \left\{\frac{(-1)^n}{n} + \frac{2}{n};\, n \in \mathbb{N}^*\right\}. \qquad (*)$$

Exercice 4 :

Using the characterization of the least upper bound and the greatest lower bound, show that :

1. $\sup A = \dfrac{3}{2}$ and $\inf A = 1$ for $A = \left\{\dfrac{3n+1}{2n+1} \mid n \in \mathbb{N}\right\}$.

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Not necessary to solve $(*)$

2. $\sup B = 2$ and $\inf B = 0$ for $B = \left\{\frac{1}{n} + \frac{1}{n^2} \mid n \in \mathbb{N}^*\right\}$.
3. $\sup C = 1$ and $\inf C = 0$ for $C = \{e^{-n} \mid n \in \mathbb{N}\}$.

*4. $\sup D = -1$ and $\inf D = -2$ for $D = \left\{\frac{1}{n^2} - 2 \mid n \in \mathbb{N}^*\right\}$.

Determine $\max A$, $\min A$, $\max B$, $\min B$, $\max C$, $\min C$, $\max D$ and $\min D$ if they exist.

Exercice 5 :

1. Show the following inequalities :
   (a) $|x| + |y| \leq |x+y| + |x-y|, \quad \forall x, y \in \mathbb{R}$.
   (b) $\sqrt{x+y} \leq \sqrt{x} + \sqrt{y}, \quad \forall x, y \in \mathbb{R}_+$.
   (c) $\left|\sqrt{x} - \sqrt{y}\right| \leq \sqrt{|x-y|}, \quad \forall x, y \in \mathbb{R}_+$.
2. Let $[x]$ denote the integer part of x. Show that for all $x, y \in \mathbb{R}$:
   (a) $x \leq y \Rightarrow [x] \leq [y]$.
   *(b) $[x] + [y] \leq [x+y] \leq [x] + [y] + 1$.

Exercice 6 :

1. Prove that :
   (a) The sum of a rational number and an irrational number is irrational.
   (b) $\sqrt{2} \notin \mathbb{Q}$.
   (c) $0.336433643364\ldots \in \mathbb{Q}$.
2. Let $a \in [1, +\infty[$. Simplify

$$x = \sqrt{a + 2\sqrt{a-1}} + \sqrt{a - 2\sqrt{a-1}}.$$

3. Compute :
   (a) $A = \sum_{k=0}^{n} \binom{n}{k}$,
   *(b) $B = \prod_{k=1}^{n} \left(1 + \frac{1}{k}\right), \quad n \in \mathbb{N}^*$.

Good Luck ! ■